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PROPAGATION OF NONLINEAR LONGITUDINAL WAVES IN POROUS SATURATED MEDIA

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A considerable number of works have been devoted to aspects of low-amplitude wave propagation in saturated porous media. A detailed bibliography of studies on this question is given in [1]. As experiments show, the upper layer of the earth's core is characterized by anomalously high values of the nonlinearity parameter [2, 3]. In view of this there is interest in questions connected with studying the propagation of finite-amplitude waves in porous media also exhibiting dispersion-dissipative properties. Nonlinear waves in a Rakhmatullin model (model of equal phase pressures) were considered in [4]. However, it is applicable to a very limited class of geological materials.

In this work a second approximation equation (KdVB) has been obtained describing propagation of longitudinal waves of finite amplitude in saturated porous media. In contrast to [4], in the model in question equality was assumed for pressure in the solid and liquid phases. Analysis of the effect of strength properties for the matrix and impregnating component on the nonlinear dissipative properties of the medium was carried out both for weakly cemented (sands) and for strongly cemented (andesite, granite) materials. Within the suggested model it is possible to describe the anomalously high values of nonlinearity parameter observed by experiment.

1. Continuity and pulse equations for the solid and liquid phases for unidimensional planar movement of a water-saturated medium have the form [1, 5]

$$\frac{\partial}{\partial t}(1-m)\rho_1+\frac{\partial}{\partial x}(1-m)\rho_1 u_1=0,\ \frac{\partial}{\partial t}m\rho_2+\frac{\partial}{\partial x}m\rho_2 u_2=0,$$
$$(1-m)\rho_1\left(\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x}\right)=-\frac{\partial p_{\text{eff}}}{\partial x}+\frac{4}{3}\frac{\partial \tau_{\text{eff}}}{\partial x}- \quad (1.1)$$
$$-(1-m)\frac{\partial p_2}{\partial x}+R+(1-m)\rho_1 g,\ m\rho_2\left(\frac{\partial u_2}{\partial t}+u_2\frac{\partial u_2}{\partial x}\right)=-m\frac{\partial p_2}{\partial x}-R+m\rho_2 g,$$

where ρ_1, ρ_2, and u_1, u_2 are solid and liquid phase density and velocity, respectively; m is medium porosity; p_2 is pore pressure; p_{eff} and τ_{eff} are effective pressure and tangential stress in the medium. Effective pressure p_{eff} is determined by the difference between pressure in the medium $p = (1 - m)p_1 + mp_2$ (p_1 is pressure in the solid phase) and pore pressure p_2:

$$p_{\text{eff}}=p-p_2=(1-m)(p_1-p_2). \quad (1.2)$$

We consider the deformation properties of a porous medium saturated with liquid. We shall assume that the difference in current porosity from porosity in the unloaded state is entirely connected with contact compressibility of particles. Whence it follows that porosity only depends on the difference of pressures in the solid phase and in the liquid saturating the pores, and there is a clear correlation between m and p_{eff}:

$$m=m(p_{\text{eff}}). \quad (1.8)$$

For dry rock $p_{\text{eff}} = p$. Therefore, the rule for the change in porosity due to effective pressure (1.3) may be determined from data for the dependence of dry rock compressibility on pressure.

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In the elastic region shear strains are determined by the shear modulus G. As follows from experiments, elasticity moduli for porous media do not depend in a linear way on pressure [6, 7]. Assuming that $G = G(p_{eff})$, we shall describe the behavior of tangential stress by Hooke's law

$$\frac{\partial \tau_{eff}}{\partial t} + u_1 \frac{\partial \tau_{eff}}{\partial x} = G(p_{eff}) \frac{\partial u_1}{\partial x}. \tag{1.4}$$

It would be correct to transpose relationships (1.3) and (1.4) with the onset of plastic acts such as infilling of pores, a dilation effect, etc.

An equation of state for each of the phases is taken in the form of a Tate equation

$$p_i = \frac{K_i}{\gamma_i}\left[\left(\frac{\rho_i}{\rho_i^0}\right)^{\gamma_i} - 1\right] \quad (i = 1, 2), \tag{1.5}$$

where K_i are volumetric compression moduli for the solid and liquid phases with $p_i = 0$; γ_i are constants governing the nonlinear properties of the phases.

Interphase friction force R in the equation for phase movement is proportional to the difference in mass velocities for the phases [1]:

$$R = \frac{m^2\mu}{k_0}(u_2 - u_1). \tag{1.6}$$

Here μ is liquid viscosity; k_0 is permeability factor for the medium.

System (1.1)-(1.6) linearized close to the initial condition describes propagation of longitudinal first and second-order waves of low amplitude in water-saturated porous media [1]. In this work only a first-order low-frequency wave is considered with characteristic frequency $\omega \ll 1/t_p$, where $t_p = k_0(1-m)\rho_1\rho_2/(m\mu\rho)$ is relaxation time for mass velocity; $\rho = (1-m)\rho_1 + m\rho_2$ is equilibrium density of the medium. It is noted that in a linear approximation system (1.1)-(1.6) is entirely equivalent to a linearized set of equations describing movement of a porous water-saturated medium obtained in [1]. In this way elasticity modulus for volumetric compression of the matrix K is connected with effective pressure p_{eff} by the relationship $K = -(1-m)\frac{dp_{eff}}{dm}\Big/\left(1 - \frac{1}{K_1}\frac{dp_{eff}}{dm}\right)$.

We expand set (1.1)-(1.6) with respect to Mach number M to the second power inclusively. In a linear approximation in a wave propagating in one direction changes of all of the values may be expressed in terms of one (e.g., in terms of the average mass velocity v). By using locally linear relationships with substitution in nonlinear terms we arrive at a nonlinear equation for v. Assuming the distortion of the wave profile caused by nonlinear and dispersion-dissipative effects only develops at distances much greater than the characteristic wavelength λ of the emitted signal, it is possible separate independent variables into "rapid" $\tau = t - x/c$ and "slow" $x(|\partial v/\partial x| \ll \frac{1}{c}|\partial v/\partial \tau|)$. By changing over to associated coordinates and discarding terms of higher orders we obtain an equation of second approximation describing propagation of first-order nonlinear waves in saturated porous media (a Korteweg—de Vries—Burgers equation):

$$\frac{\partial v}{\partial x} = \frac{\varepsilon}{c^2} v \frac{\partial v}{\partial \tau} + \eta \frac{\partial^2 v}{\partial \tau^2} - D \frac{\partial^3 v}{\partial \tau^3}. \tag{1.7}$$

The first term in the left-hand part of (1.7) describes wave distortion as a result of nonlinear effects, the second and third parts describe it as a result of viscosity and dispersion effects caused by interphase friction. Equation (1.7) is valid with the conditions $\varepsilon M \ll 1$, $ct_p \ll \lambda \ll \ell_3$ ($\ell_3 = \lambda^2/4\eta c^2$ is attenuation length of the carrier frequency of the signal). Different wave propagation regimes described by (1.7) have been studied in [8].

Sound velocity c, nonlinearity parameter ε, coefficients of viscosity η and dispersion D are expressed as follows in terms of water-saturated medium parameters:

$$c^2 = \frac{K + \frac{4}{3}G}{\rho} + \frac{K_2}{m\rho}\frac{(1 - K/K_1)^2}{1 + (1 - m - K/K_1)\frac{K_2}{mK_1}},$$

$$2\varepsilon\rho c^2 = \theta^3 K \frac{dK}{dp_{\text{eff}}} + \frac{4}{3}\theta K \frac{dG}{dp_{\text{eff}}} + 2(\varepsilon_2 - 1)\theta^3 \frac{K_2}{m^2} \times$$
$$\times \left(\frac{1 - K/K_1}{\delta}\right)^3 + 2(\varepsilon_1 - 1)(1 - m)K_1\left[\left(1 - \frac{\delta}{\sigma}\right)^3 - \left(\frac{K\theta}{(1-m)K_1}\right)^3\right] + \tag{1.8}$$
$$+ \frac{4}{3}G + 2K_2\left[1 + \frac{(1-m)\delta}{m\sigma}\left(1 + \frac{(1-m)\delta}{m\sigma} + \chi\right)\right] + 2\theta K\left[\chi - \frac{\delta^2(1 - m + (K/K_1)^2/(1 - m - K/K_1))}{2\sigma^2(1 - m - K/K_1)}\right],$$
$$\eta = \frac{t_p\alpha_1\alpha_2}{2c^5}C_{21}^2,\ D = \eta t_p\left(1 - \frac{C_{22}}{c^2} + \alpha_1\alpha_2\frac{C_{21}^2}{c^4}\right),$$

where

$$\sigma = 1 + \frac{K}{K_1}\Big/\left(1 - m - \frac{K}{K_1}\right) + \frac{1-m}{m}\frac{K_2}{K_1};\ \delta = 1 - \frac{K_2}{K_1};\ C_{21} =$$
$$= \frac{1}{\sigma\rho_1(1 - m - K/K_1)}\left[(1-m)(\rho_1/\rho_2 - 1)(1 - K/K_1)\frac{K_2}{m} - (1 - K_2/K_1)K\right] -$$
$$- \frac{4}{3}\frac{G}{(1-m)\rho_1};\ C_{22} = -\alpha_2 C_{21} + \frac{K_2}{\sigma\rho_2}[(1-m)/(1 - m - K/K_1) - \rho_2/\rho_1];$$
$$\theta = \frac{(1-m)\delta}{\sigma(1 - m - K/K_1)};\ \chi = 1 - \frac{K_2}{m\sigma K_1} - \frac{(1-m)\delta}{m^2\sigma^2}\frac{K_2}{K_1};$$

$\alpha_1 = (1 - m)\rho_1/\rho$ and $\alpha_2 = m\rho_2/\rho$ are solid and liquid phase concentrations; $\varepsilon_i = (\gamma_i + 1)/2$ ($i = 1, 2$) are phase nonlinearity parameters. In expressions (1.8) for ε the first four terms (physical nonlinearity) are specified by the nonlinear properties of the matrix, pores saturated with liquid, and solid phase particles. The rest of the terms (geometrical nonlinearity) do not make a marked contribution to the value of ε.

The nonlinearity parameter for dry rock may be obtained from (1.8) (by assuming that $K_2 = 0$) [9]: $\varepsilon_{\text{dry}} = \frac{1}{2} + \frac{1}{2}\frac{K}{K + \frac{4}{3}G}\left(\frac{dK}{dp} + \frac{4}{3}\frac{dG}{dp}\right)$. At the limit $K \ll \max\{K_1, K_2/m\}$ and $K\left(\frac{dK}{dp_{\text{eff}}} + \frac{4}{3}\frac{dG}{dp_{\text{eff}}}\right) \ll \varepsilon_2 K_2/m^2$ the model in question changes into a Rakhmatullin model which is used to describe soft soils. The nonlinearity parameter in a model of equal phase pressures was found in [4]. As follows from (1.8), it may be presented in the form $\varepsilon_p = 1 + \frac{(1-m)\delta^2}{m\sigma_0^2} + \frac{\varepsilon_2 - 1}{m\sigma_0^2} + (1-m)\frac{\varepsilon_1 - 1}{m^2\sigma_0^2}\left(\frac{K_2}{K_1}\right)^2\ \left(\sigma_0 = 1 + \frac{1-m}{m}\frac{K_2}{K_1}\right)$. With the condition $K_1 \gg \max\{K, K_2/m\}$ solid phase compressibility may be ignored and expression (1.8) takes a simple form

$$\varepsilon = \frac{K\left(\frac{dK}{dp_{\text{eff}}} + \frac{4}{3}\frac{dG}{dp_{\text{eff}}}\right) + 2\varepsilon_2\frac{K_2}{m^2} + K + \frac{4}{3}G}{2\left(K + \frac{4}{3}G + \frac{K_2}{m}\right)}.$$

Thus, Eq. (1.8) makes it possible to calculate the value of ε for saturated porous rock from the dependence of dry rock elasticity moduli on pressure.

2. We consider a weakly cemented granular material (sand type). On the basis of model representations of the Herz contact problem [10] the connection between current porosity m and porosity in the unloaded condition m_0 (structural porosity) is presented as

$$m = m_0/(1 + C\tilde{p}^n), \tag{2.1}$$

where $\tilde{p} = (p_2 - p_1)/K_1$; C and n are constants. Relationship (2.1) is written in the form

$$p_{\text{eff}} = (1 - m)K_1/C^{1/n}\left(\frac{m_0}{m} - 1\right)^{1/n}. \tag{2.2}$$

Values of C and n may be found from the change in volumetric compressibility of dry rock. In this case external pressure p is connected with $\tilde{p}$ by the relationship $p = (1 - m)K_1\tilde{p}$. By using (2.1) and considering grain compressibility we obtain an expression for volumetric deformation

$$-e = \frac{m_0 C\tilde{p}^n}{1 - m_0 + C\tilde{p}^n} + \frac{1 + C\tilde{p}^n}{1 - m_0 + C\tilde{p}^n}(1 - m_0)\tilde{p}, \tag{2.3}$$

which makes it possible to describe experimental data [6] for volumetric compressibility both for dense and for loose sand if we take C = 30.0 and n = 2/3 (from the Herz problem) with K_1 = 52.0 GPa. By using (2.2) the dependence of volumetric modulus K for weakly cemented material on effective pressure is presented in the form

$$K = K_1\left(1 + \frac{m_0(1 - m_0) nC\tilde{p}^{n-1}}{(1 - m_0 + C\tilde{p}^n)^2}\right)^{-1}, \tag{2.4}$$

whence it follows that with $\tilde{p} \to 0$ coefficient K is proportional to $\tilde{p}^{1/3}\left(K \to K_1 \frac{1 - m_0}{nm_0 C}\tilde{p}^{1/3}\right)$, and with $\tilde{p} \gg 1/C^{1/n}$, $K \to K_1$.

In the elastic region shear strains are determined by shear modulus G. Considering contact compressibility and assuming that tangential stresses are less than normal stresses, the shear modulus for sand may be approximated by the relationship

$$G^{-1} = G_1^{-1} + \frac{\zeta nC\tilde{p}^n}{(1 + C\tilde{p}^n)^2}\frac{m_0}{1 - m_0}\frac{1}{\tilde{p}} \tag{2.5}$$

(G_1 is shear modulus for a monolith; ζ is a constant determined from experimental data). By using results in [6] for the experimental dependence of Poisson's ratio on pressure, we obtain ζ = 0.63. As follows from (2.5), with small p_{eff}, $G \sim p_{eff}^{1/3}$, and with large p_{eff}, $G \to G_1$.

On the basis of relationships (1.8), (2.1), (2.4), and (2.5) numerical calculations were carried out for determining c, ε, η, and D for both weakly cemented (sands of different porosity) and for strongly cemented rocks (andesite, granite). Permeability factor for sands was calculated from the Kozeny equation [1] $k_0 = m^3 d^2/[180(1 - m)^2]$ (d is typical grain size for porous material). Calculated results for weakly cemented materials are given in Figs. 1-4 where curves 1-4 correspond dry sand with structural porosities of m_0 = 20, 30, 40, and 50%, and curves 1'-4' correspond to water-saturated sands with the same structural porosities.

Shown in Fig. 1 is the dependence of sound velocity in sands on effective pressure. As follows from (2.4) and (2.5), sound velocity in dry sand with low effective pressures is proportional to $p_{eff}^{1/6}$. Water saturation of sand markedly increases sound velocity. With effective pressures of the order to 10^{-1} MPa the sound velocity for water-saturated and dry sand differs by more than a factor of five. With an increase in pressure the sound velocity in dry sand increases rapidly and it is comparable with that in water-saturated sand with pressures of the order of 10^3 MPa.

Presented in Fig. 2 is the dependence of parameter ε on p_{eff} for different sands. It can be seen that in dry sand with effective pressures less than 1 MPa there is strong dependence of ε on pressure (as $p_{eff}^{-2/3}$) and it markedly exceeds the correspond value for water-saturated sand in which ε depends on p_{eff} much more weakly. With pressures greater than 10 MPa the value of ε for water-saturated sand becomes greater than for dry sand. Anomalously greater nonlinearity parameters ($\varepsilon \gtrsim 10^2$) for dry sand may only be observed with low effective pressures ($p_{eff} < 10^{-1}$ MPa).

Compared in Fig. 3 are the results of calculations of ε for water-saturated sand according to the model proposed and according to the Rakhmatullin model (curve 5). Since in the model of equal phase pressures p_{eff} = 0, comparison is carried out for the relationship ε(m) where in model (2.1) the nonlinearity parameter depends not only on current porosity m, but also on structural porosity m_0. As can be seen, the Rakhmatullin model makes it possible to describe quite adequately values of ε with $p_{eff} \gtrsim 1$ MPa and $m_0 \gtrsim 30\%$.

Given in Fig. 4 is the dependence of attenuation factor η in water-saturated sand on effective pressure. It can be seen from the calculations that with pressures of the order of 1 MPa there is a sharp reduction in η with an increase in effective pressure. This is mainly connected with a reduction in porosity (and consequently permeability) of the medium as a result of an increase in effective pressure. In calculating sand permeability the size of sand particles was selected as $d = 1.0 \cdot 10^{-3}$ m. In the case of quadratic attenuation for frequency the Q-factor is inversely proportional to frequency f:

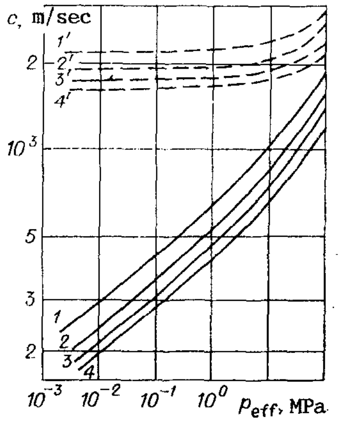

Fig. 1

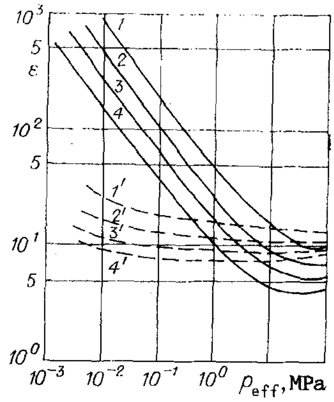

Fig. 2

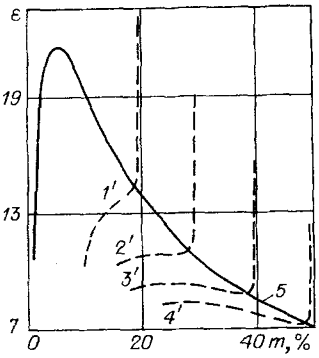

Fig. 3

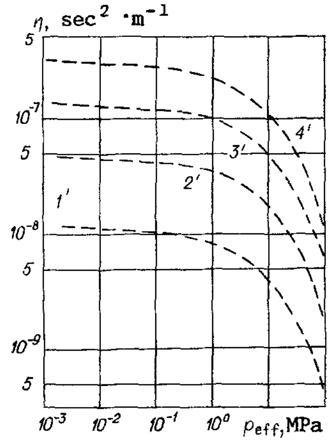

Fig. 4

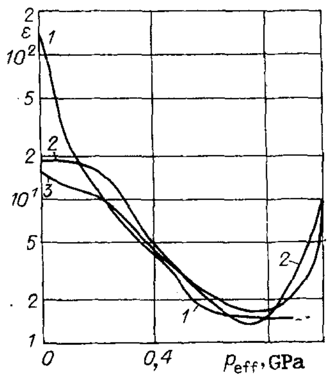

Fig. 5

$$Q = 1/(4\pi\eta cf). \tag{2.6}$$

In (2.6) the dependence of sound velocity on frequency is ignored since as calculations show the dispersion does not exceed 1-2%. For sand with an initial porosity of 40% and f = 10 Hz, with an effective pressure of the order of 10^{-1}, 10, and 10^2 MPa, Q = 40, 80, and 600, respectively. Thus, attenuation caused by interphase friction may make a marked contribution to the overall attenuation of longitudinal waves propagating in a water-saturated porous medium. However, the role of this attenuation mechanism decreases markedly with an increase in effective pressure.

Given in Fig. 5 are the results of calculating parameter ε for strongly cemented materials obtained by treating experimental dependences for elasticity moduli on pressure [7]. Curve 1 relates to Westerly granite with m_0 = 1%, 2 and 3 to dry and water-saturated andesite with porosity m_0 = 7%. The elasticity moduli derived according to pressure were calculated by means of a standard program using a fourth-order Lagrangian interpolation equation.

In order to find the dependence of nonlinearity parameter for water-saturated andesite on effective pressure on the basis of relationship (1.8) the following data were used: K_2 = 2.62 GPa, K_1 = 48.8 GPa, $\rho_2/\rho_1 = 0.4$, $\varepsilon_2 = 4$. Since in the pressure range in question equation of state (1.5) for the solid phase may be linearized, in the calculations it was assumed that $\varepsilon_1 = 1$.

As follows from Fig. 5, the value of ε with small p_{eff} for strongly cemented rocks may reach the order of 10^2. In contrast to weakly cemented rocks the nonlinearity parameter for material with $K/K_1 \sim 1$ depends weakly on water saturation. It can be seen from the calculations that attenuation caused by interphase friction is markedly less than that observed by experiment for strongly cemented rocks.

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SOLUTION OF A NONSTATIONARY PROBLEM OF ELASTICITY THEORY

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UDC 539.3

In this paper we present a new approach to the solution of nonstationary anti-plane boundary value problems of linear elasticity theory for semi-bounded regions of the type of a halfspace or a layer with mixed boundary conditions both on their surfaces (systems of stamps) and also their interiors (cracks, inclusions). Application of an additional integral Laplace transform with respect to the time for reducing the above-named boundary value problems to the solution of an integral equation gives rise to certain difficulties in its solution in comparison with problems of stationary oscillations, methods for the solution of which are, at the present time, well worked-out. The majority of processes, however, are essentially of a nonstationary nature and cannot be reduced to problems of harmonic analysis. The solution, therefore, of nonstationary problems calls for urgent attention.

According to the method we propose, using properties of the inversion of Laplace and Fourier convolutions of two functions, the initial boundary value problem can be reduced to the solution of a Volterra integral equation of the first kind for the unknown function itself and not its integral transform. In this connection, the Laplace and Fourier transforms are carried over with the unknown function onto the kernel, which is given by analytic expression in explicit form. The original of this kernel is then found by Cagniard's method

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